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Moments, Characteristic and Moment Generating Function of a Three Parameter Weibul Distribution – an Explicit Independent Approach

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ABSTRACT
It is obvious that the characteristic function of three-parameter Weibull distribution can be derived independently in an explicit form and hence the moment generating function is then deduced from it. In this paper, the mean and variance were derived independently. The expression of the mean, variance, skewness and kurtosis are also gotten from the Moment Generating Function (MGF)

Key words: Moment generating function, Three-parameter Weibull distribution, Characteristic function, Skewness, Kurtosis

THE WEIBULL DISTRIBUTION

The weibull distribution has been widely applied to many random phenomena. The principal utility of the weibul distribution is that it affords an excellent approximation to the probability law of many random variables (William et al; 2003). The probability density function of three-parameter weibull distribution is given by

$$f(x)=\frac{\lambda}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\lambda-1}\ \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right],\ \mu<x<\infty \qquad \ldots \qquad (1)$$

Where, $\mu,\sigma,\lambda>0,$ are respectively the location, scale and shape parameters of the distribution? Its cumulative distribution function is given by

$$F(x)=1-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right] \qquad \ldots \qquad (2)$$

Mean of the Weibull Distribution by definition, of a continuous random variable X is given by;

$$E(X)=\int_{-\infty}^{\infty}xf(x)dx \qquad \ldots \qquad (3)$$

$$=\int_{\mu}^{\infty} x\frac{\lambda}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\lambda-1}\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$\int_{\mu}^{\infty} x\frac{\lambda}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\left(\frac{x-\mu}{\sigma}\right)^{-1}\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx \qquad \ldots \qquad (4)$$

Using integral by algebraic substitution, we have;

Let $w=\left(\frac{x-\mu}{\sigma}\right)^{\lambda} \Rightarrow w^{\frac{1}{\lambda}}=\frac{x-\mu}{\sigma}$

$$\sigma w^{\frac{1}{\lambda}}=x-\mu, \qquad \therefore x=\sigma w^{\frac{1}{\lambda}}+\mu$$

$$\frac{dx}{dw}=\frac{\sigma}{\lambda}w^{\frac{1-\lambda}{\lambda}}, \qquad dx=\frac{\sigma}{\lambda}w^{\frac{1-\lambda}{\lambda}}dw$$

Substitute x and dx into equation (4) to get

$$E(x)=\frac{\lambda}{\sigma}\int_{\mu}^{\infty}\left(\sigma w^{\frac{1}{\lambda}}+\mu\right)w\left(\frac{\sigma w^{\frac{1}{\lambda}}+\mu-\mu}{\sigma}\right)^{-1}\exp[-w]\left(\frac{\sigma}{\lambda}w^{\frac{1-\lambda}{\lambda}}dw\right)$$

$$=\frac{\lambda}{\sigma}\int_{\mu}^{\infty}\left(\sigma w^{\frac{1}{\lambda}}+\mu\right)w\left(w^{\frac{1}{\lambda}}\right)^{-1}\exp(-w)\frac{\sigma}{\lambda}w^{\frac{1-\lambda}{\lambda}}dw=\int_{\mu}^{\infty}\left(\sigma w^{\frac{1}{\lambda}}+\mu\right)(w)^{1-\frac{1}{\lambda}}\exp(-w)w^{\frac{1}{\lambda}-1}dw$$

$$=\int_{\mu}^{\infty}\sigma w^{\frac{1}{\lambda}}\exp(-w)dw+\int_{\mu}^{\infty}\mu\exp(-w)dw=\sigma\int_{\mu}^{\infty}w^{\frac{1}{\lambda}+1-1}\exp(-w)dw+\mu\int_{\mu}^{\infty}\exp(-w)dw$$

$$=\mu+\sigma\Gamma\left(1+\frac{1}{\lambda}\right)$$

$$\therefore E(x)=\mu+\sigma\Gamma\left(1+\frac{1}{\lambda}\right) \qquad \ldots \qquad (5)$$

The variance of a continuous random variable is defined as;

$$\operatorname{var}(X)=E\left(X^2\right)-[E(X)]^2 \qquad \ldots \qquad (6)$$

But $E\left(X^2\right)=\int_{-\infty}^{\infty}x^2 f(x)dx$

$$= \int_{\mu}^{\infty} x^2 \frac{\lambda}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\lambda-1} \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \frac{\lambda}{\sigma}\int_{\mu}^{\infty} x^2\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\left(\frac{x-\mu}{\sigma}\right)^{-1} \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx \quad ..(7)$$

Let $p = \left(\frac{x-\mu}{\sigma}\right)^{\lambda}$, $p^{\frac{1}{\lambda}} = \left(\frac{x-\mu}{\sigma}\right)$

$$\sigma p^{\frac{1}{\lambda}} = x-\mu, \qquad \therefore x = \mu + \sigma p^{\frac{1}{\lambda}}$$

$$\frac{dx}{dp} = \frac{\sigma}{\lambda} p^{\frac{1}{\lambda}-1}, \qquad \therefore dx = \frac{\sigma}{\lambda} p^{\frac{1}{\lambda}-1} dp$$

Substitute x and dp into equation (7) to get

$$E\left(x^2\right) = \frac{\lambda}{\sigma}\int_{\mu}^{\infty}\left(\mu + \sigma p^{\frac{1}{\lambda}}\right)^2 p\left(\frac{\mu + \sigma p^{\frac{1}{\lambda}} - \mu}{\sigma}\right)^{-1} \exp[-p]\frac{\sigma}{\lambda} p^{\frac{1}{\lambda}-1} dp$$

$$\int_{\mu}^{\infty}\left(\mu^2 + 2\mu\sigma p^{\frac{1}{\lambda}} + \sigma^2 p^{\frac{2}{\lambda}}\right) p\left(p^{-\frac{1}{\lambda}}\right)\exp[-p] p^{\frac{1}{\lambda}-1} dp$$

$$\int_{\mu}^{\infty}\left(\mu^2 + 2\mu\sigma p^{\frac{1}{\lambda}} + \sigma^2 p^{\frac{2}{\lambda}}\right)\exp[-p]dp$$

$$\int_{\mu}^{\infty}\mu^2 \exp(-p)dp + \int_{\mu}^{\infty} 2\mu\sigma p^{\frac{1}{\lambda}} \exp(-p)dp + \int_{\mu}^{\infty}\sigma^2 p^{\frac{2}{\lambda}} \exp(-p)dp$$

$$\mu^2\int_{\mu}^{\infty}\exp(-p)dp + 2\mu\sigma\int_{\mu}^{\infty} p^{\frac{1}{\lambda}+1-1} \exp(-p)dp + \sigma^2\int_{\mu}^{\infty} p^{\frac{2}{\lambda}+1-1} \exp(-p)dp$$

$$= \mu^2 + 2\mu\sigma\Gamma\left(\frac{1}{\lambda}+1\right) + \sigma^2\Gamma\left(\frac{2}{\lambda}+1\right)$$

$$\therefore E\left(x^2\right) = \mu^2 + 2\mu\sigma\Gamma\left(1+\frac{1}{\lambda}\right) + \sigma^2\Gamma\left(1+\frac{2}{\lambda}\right) \qquad \ldots \qquad (8)$$

Substitute Equations (5) and (8) into equation (6) to get;

$$\operatorname{var}(X)=\mu^2+2\mu\sigma\Gamma\left(1+\frac{1}{\lambda}\right)+\sigma^2\Gamma\left(1+\frac{2}{\lambda}\right)-\mu^2-2\mu\sigma\Gamma\left(1+\frac{1}{\lambda}\right)-\sigma^2\left[\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2$$

$$=\sigma^2\Gamma\left(1+\frac{1}{\lambda}\right)-\sigma^2\left[\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2=\sigma^2\left\{\Gamma\left(1+\frac{2}{\lambda}\right)-\left[\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2\right\}$$

Characteristic function of three parameter weibull Distribution

The characteristic function of three-parameter weibull distribution is given by;

$$\phi(t)=E[\exp(itx)]=\int_{\mu}^{\infty}\exp(itx)f(x)dx$$

But $\exp(itx)=\cos tx+i\sin tx$

$$\therefore\phi(t)=\int_{\mu}^{\infty}\cos(tx)f(x)dx+i\int_{\mu}^{\infty}\sin(tx)f(x)dx$$

Let $A=\int_{\mu}^{\infty}\cos(tx)f(x)dx$ and $B=i\int_{\mu}^{\infty}\sin(tx)f(x)dx$

Solving A using integration by parts, we have

$$\int udv=uv-\int vdu$$

$$u=\cos(tx),\frac{du}{dx}=-t\sin(tx),du=-t\sin(tx)dx$$

$$dv=f(x)dx,\therefore\int dv=\int f(x)dx$$

i.e $v=-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]$

$$\therefore A=[-\cos(tx)]\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]_{\mu}^{\infty}-t\int_{\mu}^{\infty}\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]\sin(tx)dx$$

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$$= -\cos(t\infty)\exp\left[-\left(\frac{\infty-\mu}{\sigma}\right)^{\lambda}\right] + \cos(t\mu)\exp\left[-\left(\frac{\mu-\mu}{\sigma}\right)^{\lambda}\right] - \int_{\mu}^{\infty} t\sin(tx)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$\therefore A = \cos(t\mu) - \int_{\mu}^{\infty} t\sin(tx)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

Solving B using integration by parts, we get

$$u = i\sin(tx), \frac{du}{dx} = it\cos(tx), du = it\cos(tx)dx$$

$$\therefore B = \left[-i\sin(tx)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]\right]_{\mu}^{\infty} + \int_{\mu}^{\infty} it\cos(tx)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$\therefore B = i\sin(t\mu) + \int_{\mu}^{\infty} it\cos(tx)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$A + B = \int_{\mu}^{\infty}[\cos(tx) + i\sin(tx)]f(x)dx$$

$$= \cos(t\mu) + i\sin(t\mu) + \int_{\mu}^{\infty}(it\cos(tx) - t\sin(tx))\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \exp(it\mu) + \int_{\mu}^{\infty}(-i^{3}t\cos(tx) + i^{2}t\sin(tx))\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \exp(it\mu) + \int_{\mu}^{\infty}(i\ t\cos(tx) + i^{2}t\sin(tx))\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \exp(it\mu) + it\int_{\mu}^{\infty}(\cos(tx) + i\sin(tx))\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \exp(it\mu) + it\int_{\mu}^{\infty}\exp(itx)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

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$$= \exp(it\mu) + it\int_{\mu}^{\infty}\left(1 + itx + \frac{(itx)^2}{2!} + \frac{(itx)^3}{3!} + \ldots\right)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \exp(it\mu) + \int_{\mu}^{\infty}\left(it + (i\ t)^2 x + \frac{(i\ t)^3 x^2}{2!} + \frac{(i\ t)^4 x^3}{3!}\ldots\right)\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

$$= \exp(it\mu) + \sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\int_{\mu}^{\infty}x^r \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\lambda}\right]dx$$

Let $p = \left(\frac{x-\mu}{\sigma}\right)^{\lambda}$, $\quad p^{\frac{1}{\lambda}} = \frac{x-\mu}{\sigma}$, $\quad \sigma p^{\frac{1}{\lambda}} = x - \mu$

$\therefore x = \mu + \sigma p^{\frac{1}{\lambda}}$, $\quad \frac{dx}{dp} = \frac{\sigma}{\lambda}p^{\frac{1}{\lambda}-1}$, $\quad dx = \frac{\sigma}{\lambda}p^{\frac{1}{\lambda}-1}dp$

$$\therefore \phi(t) = \exp(it\mu) + \sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\int_{\mu}^{\infty}\left(\mu + \sigma p^{\frac{1}{\lambda}}\right)^r \exp[-p]\frac{\sigma}{\lambda}p^{\frac{1}{\lambda}-1}dp$$

$$= \exp(it\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\int_{\mu}^{\infty}\left(\mu + \sigma p^{\frac{1}{\lambda}}\right)^r \exp[-p]p^{\frac{1-\lambda}{\lambda}}dp$$

$$= \exp(it\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\int_{\mu}^{\infty}\left[\binom{r}{0}\mu^r\left(\sigma p^{\frac{1}{\lambda}}\right)^0 + \binom{r}{1}\mu^{r-1}\left(\sigma p^{\frac{1}{\lambda}}\right) + \binom{r}{2}\mu^{r-2}\left(\sigma p^{\frac{1}{\lambda}}\right)^2 + \ldots + \binom{r}{r}\mu^{r-r}\left(\sigma p^{\frac{1}{\lambda}}\right)^r\right]\exp(-p)p^{\frac{1}{\lambda}-1}dp$$

$$= \exp(it\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\mu^r\int_{\mu}^{\infty}\left[\binom{r}{0}\left(\frac{\sigma}{\mu}\right)^0\left(p^{\frac{1}{\lambda}}\right)^0 + \binom{r}{1}\left(\frac{\sigma}{\mu}\right)^1\left(p^{\frac{1}{\lambda}}\right)^1 + \binom{r}{2}\left(\frac{\sigma}{\mu}\right)^2\left(p^{\frac{1}{\lambda}}\right)^2 + \ldots + \binom{r}{r}\left(\frac{\sigma}{\mu}\right)^r\right]\exp(-p)p^{\frac{1}{\lambda}-1}dp$$

$$= \exp(it\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\mu^r\left\{\begin{array}{l}\binom{r}{0}\left(\frac{\sigma}{\mu}\right)^0\int_{\mu}^{\infty}\exp(-p)p^{\frac{1}{\lambda}-1}dp + \binom{r}{1}\left(\frac{\sigma}{\mu}\right)\int_{\mu}^{\infty}\exp(-p)p^{\frac{2}{\lambda}-1}dp + \binom{r}{2}\left(\frac{\sigma}{\mu}\right)^2\int_{\mu}^{\infty}\exp(-p)p^{\frac{3}{\lambda}-1}dp + \ldots \\ \binom{r}{r}\left(\frac{\sigma}{\mu}\right)^r\int_{\mu}^{\infty}\exp(-p)p^{\frac{r}{\lambda}-1}dp\end{array}\right\}$$

$$= \exp(it\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\frac{(it)^{r+1}}{r!}\mu^r\left\{\binom{r}{0}\left(\frac{\sigma}{\mu}\right)^0\Gamma\left(\frac{1}{\lambda}\right) + \binom{r}{1}\left(\frac{\sigma}{\mu}\right)^1\Gamma\left(\frac{2}{\lambda}\right) + \binom{r}{2}\left(\frac{\sigma}{\mu}\right)^2\Gamma\left(\frac{3}{\lambda}\right) + .. + \binom{r}{r}\left(\frac{\sigma}{\mu}\right)^r\Gamma\left(\frac{r}{\lambda}\right)\right\}$$

$$= \exp\ (it\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\left(\frac{it}{r!}\right)^{r+1}\mu^r\sum_{n=0}^{r}\binom{r}{n}\left(\frac{\sigma}{\mu}\right)^n\Gamma\left(\frac{n+1}{\lambda}\right), r = 0,1,2...$$

Where $\binom{r}{n} = \frac{r!}{n!(r-n)!}$

$$= \left(1 + \frac{it\mu}{1!} - \frac{t^2\mu^2}{2!} - ...\right) + \frac{\sigma}{\lambda}\left\{\begin{array}{l} it\Gamma\left(\frac{1}{\lambda}\right) - t^2\mu\left[\Gamma\left(\frac{1}{\lambda}\right) + \left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right) - \right] \\ \frac{it^3}{2}\mu^2\left[\left(\frac{1}{\lambda}\right) + 2\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right) + \left(\frac{\sigma}{\mu}\right)^2\Gamma\left(\frac{3}{\lambda}\right) + ...\right.\end{array}\right\}$$

or

$$\phi(t) = \sum_{r=0}^{\infty}\frac{(it\sigma)^r}{r!} \qquad r\left(1 + \frac{\sigma}{\lambda}\right)$$

Moment Generating Function of a three Parameter Weibull Distribution

The moment generating function can be easily gotten from the characteristics functions. That is by arranging $it = \phi$,

$$M(\phi) = \exp(\theta\mu) + \frac{\sigma}{\lambda}\sum_{r=0}^{\infty}\frac{(\theta)^{r+1}}{r!}\mu^r\sum_{n=0}^{r}\binom{r}{n}\left(\frac{\sigma}{\mu}\right)^n\Gamma\left(\frac{n+1}{\lambda}\right) \qquad \dots (10)$$

Equation (10) is the moment generating function of the three parameter weibull distribution. Thus we can generate all the raw moments of the distribution. That is

$$M_X^{(n)}(0) = \mu_n^1 = E(X^n) = \left[\left(\frac{d^n(M(\theta))}{d\theta^n}\right)\right]_{\theta=0} \qquad \dots (11)$$

$$M(\theta)=\left(1+\frac{\theta\mu}{1!}+\frac{\theta^2\mu^2}{2!}+...\right)+\frac{\sigma}{\lambda}\left\{\begin{array}{l}\theta\left[\Gamma\left(\frac{1}{\lambda}\right)\right]+\theta^2\mu\left[\Gamma\left(\frac{1}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)\right]+\frac{\theta^3\mu^2}{2!}\\ \left[\Gamma\left(\frac{1}{\lambda}\right)+2\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)^2\Gamma\left(\frac{3}{\lambda}\right)\right]+\frac{\theta^4\mu^3}{3!}\\ \left[\Gamma\left(\frac{1}{\lambda}\right)+3\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{3}{\lambda}\right)+2\left(\frac{\sigma}{\mu}\right)^2\Gamma\left(\frac{2}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{1}{\lambda}\right)\right]+...\end{array}\right\}$$

$$M_X^i(\theta)=\left(\mu+\frac{2\theta\mu^2}{2!}+...\right)+\frac{\sigma}{\lambda}\left\{\begin{array}{l}\Gamma\left(\frac{1}{\lambda}\right)+2\theta\mu\left[\Gamma\left(\frac{1}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)\right]+\frac{3\theta^2\mu^2}{2!}\\ \left[\Gamma\left(\frac{1}{\lambda}\right)+2\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)^2\Gamma\left(\frac{3}{\lambda}\right)\right]+...\end{array}\right\}$$

$$\therefore Mean=M_X^i(0)=\mu_1' \;=\mu+\frac{\sigma}{\lambda}\left\{\Gamma\left(\frac{1}{\lambda}\right)\right\}\;=\mu+\frac{\sigma}{\lambda}\left\{\Gamma\left(\frac{1}{\lambda}\right)\right\}$$

$$\therefore Mean=\mu+\sigma\Gamma\left(1+\frac{1}{\lambda}\right)$$

$$M_X^{ii}(\theta)=\left(\mu^2+...\right)+\frac{\sigma}{\lambda}\left\{2\mu\left[\Gamma\left(\frac{1}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)\right]+\frac{6\theta\mu^2}{2!}\left[\Gamma\left(\frac{1}{\lambda}\right)+2\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)^2\Gamma\left(\frac{3}{\lambda}\right)\right]+...\right\}$$

$$\therefore M_X^{ii}(0)=\mu_2' \;=\mu^2+\frac{\sigma}{\lambda}\left\{2\mu\left[\Gamma\left(\frac{1}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)\Gamma\left(\frac{2}{\lambda}\right)\right]\right\}$$

$$=\mu^2+\frac{\sigma}{\lambda}\left\{2\mu\left[\lambda\Gamma\left(1+\frac{1}{\lambda}\right)+\left(\frac{\sigma}{\mu}\right)\frac{\lambda}{2}\Gamma\left(1+\frac{2}{\lambda}\right)\right]\right\}$$

$$=\mu^2+\frac{\sigma}{\lambda}\left\{2\mu\lambda\Gamma\left(1+\frac{1}{\lambda}\right)+\sigma\lambda\Gamma\left(1+\frac{2}{\lambda}\right)\right\}=\mu^2+2\sigma\mu\Gamma\left(1+\frac{1}{\lambda}\right)+\sigma^2\Gamma\left(1+\frac{2}{\lambda}\right)$$

$$Var(X)=\mu_2' -\left(\mu_1'\right)^2=\mu^2+2\sigma\mu\Gamma\left(1+\frac{1}{\lambda}\right)+\sigma^2\Gamma\left(1+\frac{2}{\lambda}\right)-\left[\mu+\sigma\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2$$

$$= \mu^2 + 2\sigma\mu\Gamma\left(1+\frac{1}{\lambda}\right) + \sigma^2\Gamma\left(1+\frac{2}{\lambda}\right) - \left[\mu^2 + 2\mu\sigma\Gamma\left(1+\frac{1}{\lambda}\right) + \sigma^2\left[\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2\right]$$

$$= \mu^2 + 2\sigma\mu\Gamma\left(1+\frac{1}{\lambda}\right) + \sigma^2\Gamma\left(1+\frac{2}{\lambda}\right) - \mu^2 - 2\mu\sigma\Gamma\left(1+\frac{1}{\lambda}\right) - \sigma^2\left[\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2$$

$$= \sigma^2\Gamma\left(1+\frac{2}{\lambda}\right) - \sigma^2\left[\Gamma\left(1+\frac{1}{\lambda}\right)\right]^2$$

$$\therefore Var(X) = \sigma^2\left[\Gamma\left(1+\frac{2}{\lambda}\right) - \left(\Gamma\left(1+\frac{1}{\lambda}\right)\right)^2\right]$$

$\mu_n - n^{th}$ arbitrary moment or nth raw moment. Accordingly

$$\text{Mean } = \mu_1 = \mu + \sigma\Gamma\left(1+\frac{1}{\lambda}\right) \quad \ldots \ (12)$$

$$Var(x) = \mu_2 = \mu_2 - (\mu_1)^2 = \sigma^2\left[\Gamma\left(1+\frac{2}{\lambda}\right) - \Gamma^2\left(1+\frac{1}{\lambda}\right)\right] \quad \ldots \ (13)$$

$$\text{Skewness } = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} = \frac{g_3 - 3g_1g_2 + 2g_1^3}{(g_2 - g_1^2)^{\frac{3}{2}}} \quad \ldots (14)$$

$$\text{Kurtosis } = \frac{\mu_4}{(\mu_2)^2} = \frac{g_4 - 4g_1g_3 + 6g_2g_1^2 - 3g_1^4}{(g_2 - g_1^2)^2}$$

Conclusion

This paper has been able to identify the three parameter Weibull distribution which is widely used in extreme event modeling. Hence, its *CF* is derived and it satisfies all the conditions for a function to be a characteristic function. It is able to generate all the moments of 3-parameter *Weibull* distribution from its *MGF*. Thus, expressions for mean, variance, skewness and kurtosis are also gotten from *MGF*.

References

[1] Kendall G.M. and Stuart A.,(1964). The Advanced Theory of Statistics; Distribution Theory, 1, London: Charles Griffin and Co.Ltd., London **(1964)**

[2] Muraleedharan G., Rao, A.D., Kurup P.G., Unnikrishnan Nair N. and Mourani S., (2007). Modified Weibull Distribution for Maximum and Significant Wave Height Simulation and Prediction, *Coast. Engng.,* **54**(**8**)**,** 630-638 (**2007**)

[3] Muraleedharan G., Unnikrishnan Nair N. and Kurup P.G.,(1999). Application of Weibull model Redefined Significant Wave Height Distributions, *Proc. Ind. Aca. Sci.,Ear. Planet. Sci.,* **108 (3)**, 149-153 (**1999**)

[4] Ochi M.,(1998) Ocean Waves, Ocean Technology Series-6; Cambridge University Press: Cambridge, UK (1998)